Study Guide

Direct Variation Quiz
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Direct Variation
Variation equations are formulas that show how one quantity changes in relation to one or more other quantities. There are four types of variation: direct, indirect (or inverse), joint, and combined.

Direct variation equations show a relationship between two quantities such that when one quantity increases, the other also increases, and when one quantity decreases, the other also decreases. We can say that *y* varies directly as *x*, or *y* is proportional to *x*. Direct variation formulas are of the form $y = kx$, where the number represented by *k* does not change and is called a constant of variation.

Indirect variation equations are of the form $y = k/x$ and show a relationship between two quantities such that when one quantity increases, the other decreases, and vice versa.

This skill focuses on direct variation. The following is an example of a direct variation problem.

The amount of money in a paycheck, *P*, varies directly as the number of hours, *h*, that are worked. In this case, the constant *k* is the hourly wage, and the formula is written $P = kh$. If the equation is solved for *k*, the resulting equation shows that *P* and *h* are proportional to each other.
$k = \frac{P}{h}$ Therefore, when two variables show a direct variation relationship, they are proportional to each other. Direct variation problems can be solved by setting up a proportion in the form below.

$$\frac{P_1}{h_1} = \frac{P_2}{h_2}$$

P_1 = the amount of the first paycheck
h_1 = the number of hours worked for the first paycheck
P_2 = the amount of the second paycheck
h_2 = the number of hours worked for the second paycheck

Example 1: The amount of fuel needed to run a textile machine varies directly as the number of hours the machine is running. If the machine required 8 gallons of fuel to run for 24 hours, how many gallons of fuel were needed to run the machine for 72 hours? Round your answer to the nearest tenth of a gallon, if necessary.

(1) $\frac{8 \text{ gallons}}{24 \text{ hours}} = \frac{g \text{ gallons}}{72 \text{ hours}}$

(2) $\frac{8 \text{ gallons}}{24 \text{ hours}} \times \frac{g \text{ gallons}}{72 \text{ hours}}$

(3) $(g \text{ gallons})(24 \text{ hours}) = (8 \text{ gallons})(72 \text{ hours})$

(4) $\frac{(g \text{ gallons})(24 \text{ hours})}{(24 \text{ hours})} = \frac{(8 \text{ gallons})(72 \text{ hours})}{(24 \text{ hours})}$

(5) $\frac{(g \text{ gallons})\cancel{(24 \text{ hours})}}{\cancel{(24 \text{ hours})}} = \frac{(8 \text{ gallons})\cancel{(72 \text{ hours})}^{3}}{\cancel{(24 \text{ hours})}_{1}}$

(6) $g = 24 \text{ gallons}$

Step 1: Set up the proportion. Since the machine used 8 gallons of fuel in 24 hours, the left side of the proportion should be 8 gallons over 24 hours. The number of gallons that the machine used in 72 hours needs to be found, so the right side of the proportion should be *g* gallons over 72 hours.
Step 2: Cross-multiply across the equal sign.
Step 3: Set up the cross-multiplication equation.
Step 4: Divide both sides of the equation by 24 hours to isolate *g* gallons.
Step 5: Reduce the fractions on both sides of the equal sign.

Step 6: Simplify by multiplying the numbers remaining on the right side of the equal sign (8 gallons $\times$ 3).

Answer: 24 gallons

Example 2: The price of jellybeans, *j*, varies directly as the number of pounds, *p*, that are purchased. Find the equation that relates the two variables if jellybeans are $1.95 per pound.

(1) $y = kx, j = kp$
(2) $j = 1.95p$

Step 1: Remember that the formula for direct variation is: $y = kx$ and substitute the variables from the question into the appropriate places.
Step 2: Since the jellybeans are always $1.95 per pound, the constant, *k*, equals 1.95. Substitute 1.95 into the equation for *k*.

Answer: $j = 1.95p$

Activities that can help reinforce the concept of direct variation are as follows.

1. Have students solve the equation $y = kx$ for *k*, and then substitute two sets of (x, y) values into the equation and compare the values for *k*. If they are the same, then *x* and *y* have a direct variation relationship.
2. Have the student think of scenarios that show a direct variation relationship. Then, make up numbers to go with the relationships and have the students practice solving them.